# Prospects for a superradiant laser

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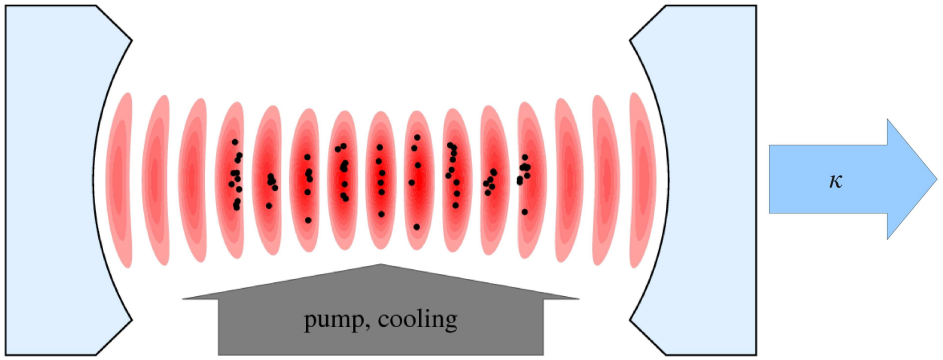
Kioloa Workshop

D. Meiser, Jun Ye, D. Carlson, and MH, PRL **102**, 163601 (2009).
D. Meiser and MH, PRA **81**, 033847 (2010).
D. Meiser and MH, PRA 81, 063827 (2010).

# Overview

- Conventional laser: What do we mean by a laser?
- The basic idea: Superradiance in steady-state
- Application with $^{87}\mathrm{Sr}$: Making a stable coherent light source

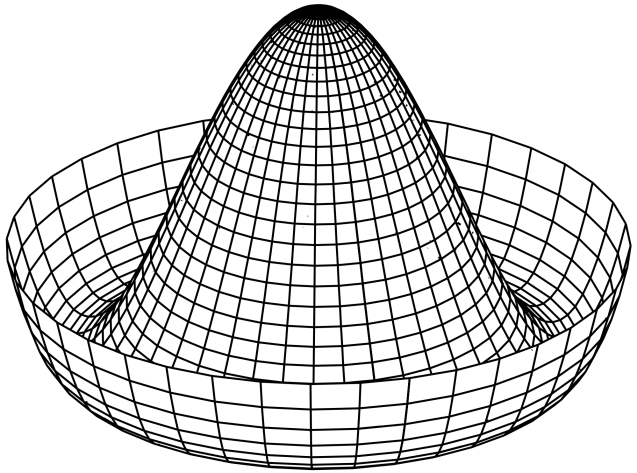
# Conventional laser




- Cavity, internal ratchet, pumping, output coupling, . . .
- Brightness; many photons per mode
- Coherent; long coherence length, coherence time

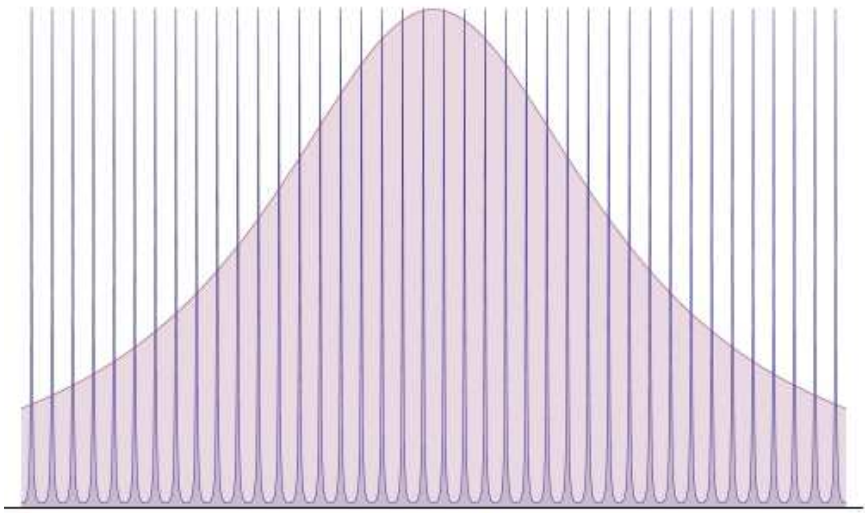
# Schawlow-Townes Linewidth

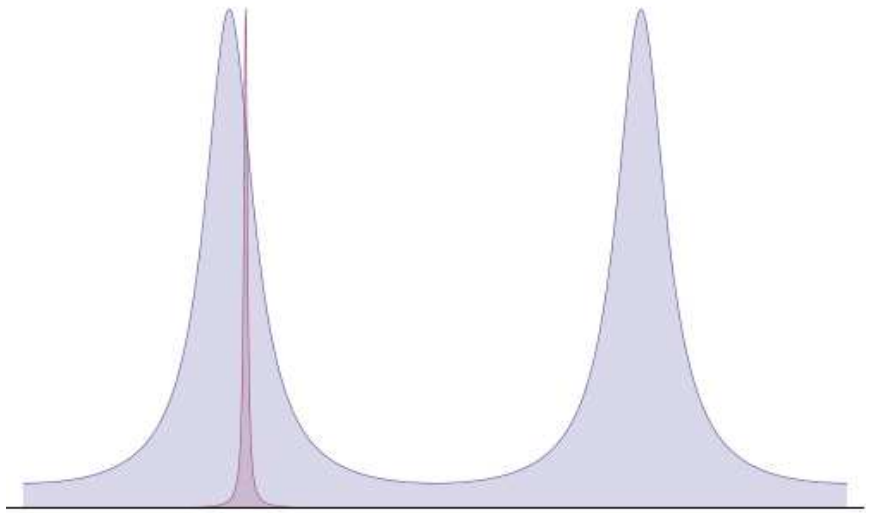
Noise added to circulating field
Amplitude fluctuations damped
Phase fluctuations→random walk



# Where are the atom dynamics?

| | |
|---|---|
| $\Gamma$ | Atomic decay rate |
| $\kappa$ | Cavity decay rate |
| $g$ | Atom-cavity coupling |

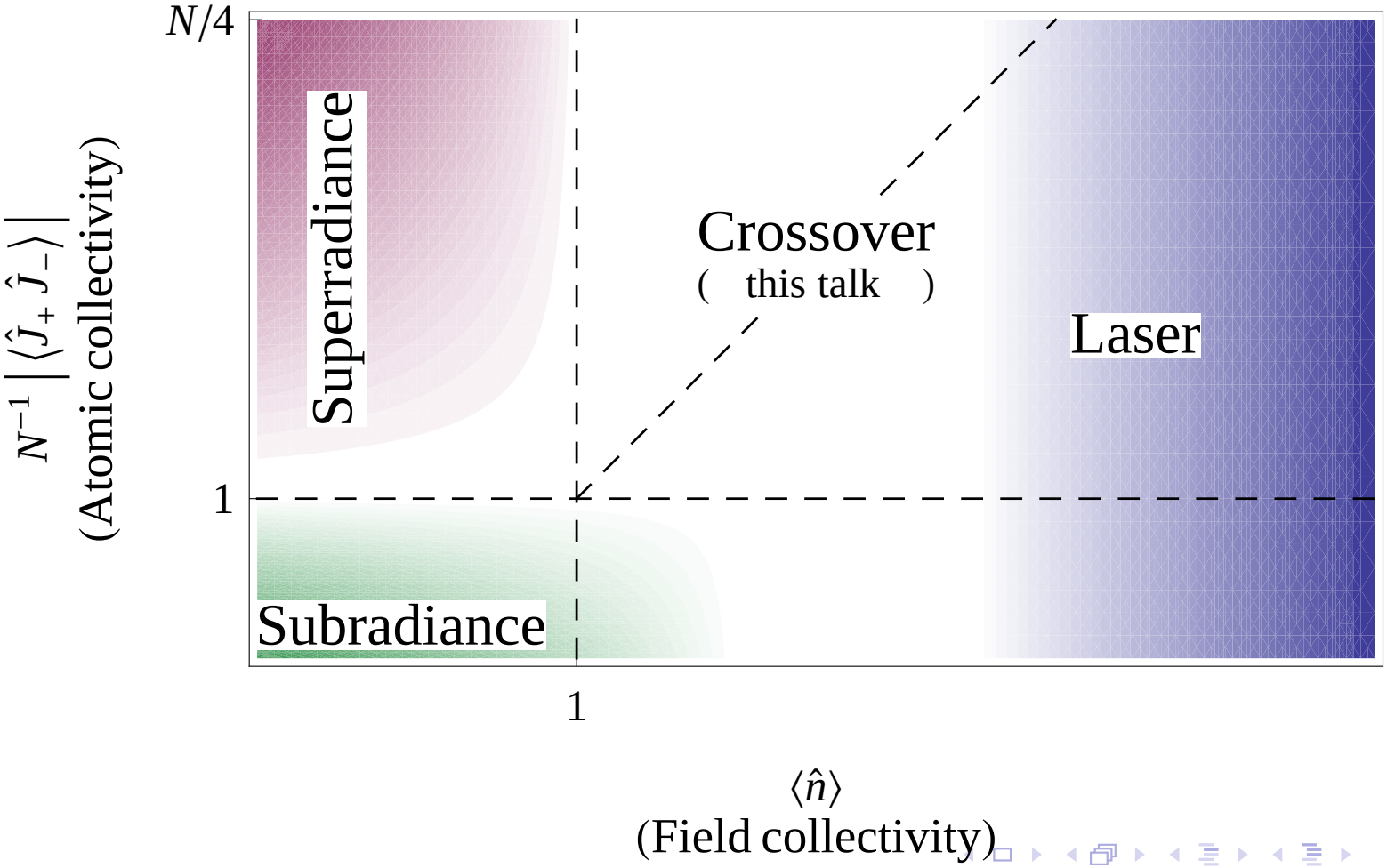




- Normal laser: atomic relaxation rates much faster than field relaxation rates (left case)
- Cavity dynamics completely “contained” in bandwidth of atom
- Adiabatic elimination of the atoms: $\Gamma \gg \kappa, g$

# Basic idea: reverse the roles of atoms and cavity

- $n$ cavity photon occupancy: importance of stimulated emission
- $N^{-1}|\left\langle \hat{J}_+\hat{J}_- \right\rangle|$, with N the atom number: importance of superradiant emission



# Superradiance: most simple example

- Two atoms, two internal states $\uparrow$ and $\downarrow$
- Decay operator $J_- = \sqrt{\Gamma}\left(\sigma_-^{(1)} + \sigma_-^{(2)}\right)$
- Emission rate $\langle\psi|J_+J_-|\psi\rangle$

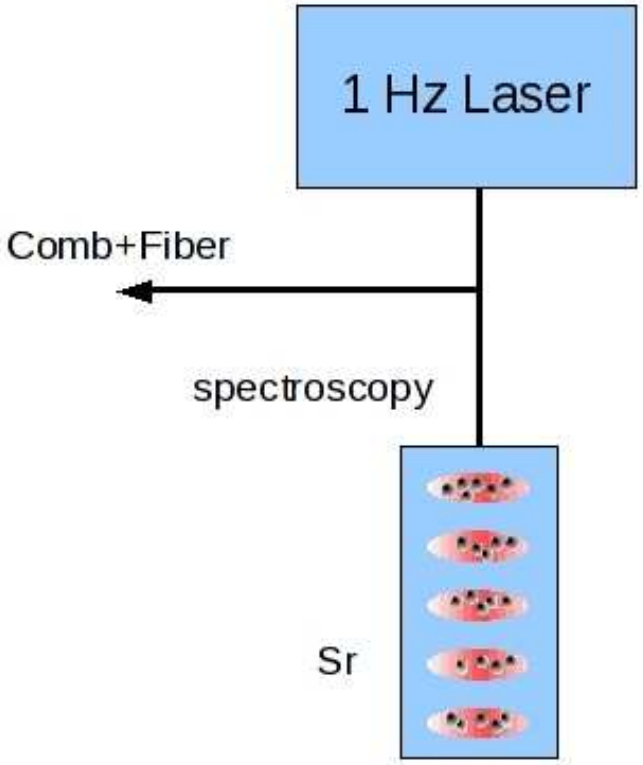
**Four basic possibilities for $|\psi\rangle$**

- $|\uparrow\uparrow\rangle$, emission rate $2\Gamma$
- $\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)$, emission rate $2\Gamma$, superradiance
- $\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$, emission rate $0$, subradiance
- $|\downarrow\downarrow\rangle$, emission rate $0$

$\rightarrow$ Constructive or destructive quantum interference

# How does optical atomic clock work? (theorist's version)

- Frequency standard: Ultra-narrow atomic transition (e.g. in Sr, $\sim 1$ mHz)
- Counter: Ultra-stable laser ($\lesssim 1$ Hz) + frequency comb
- Bottleneck: Interrogation laser



# Circumventing reference cavity

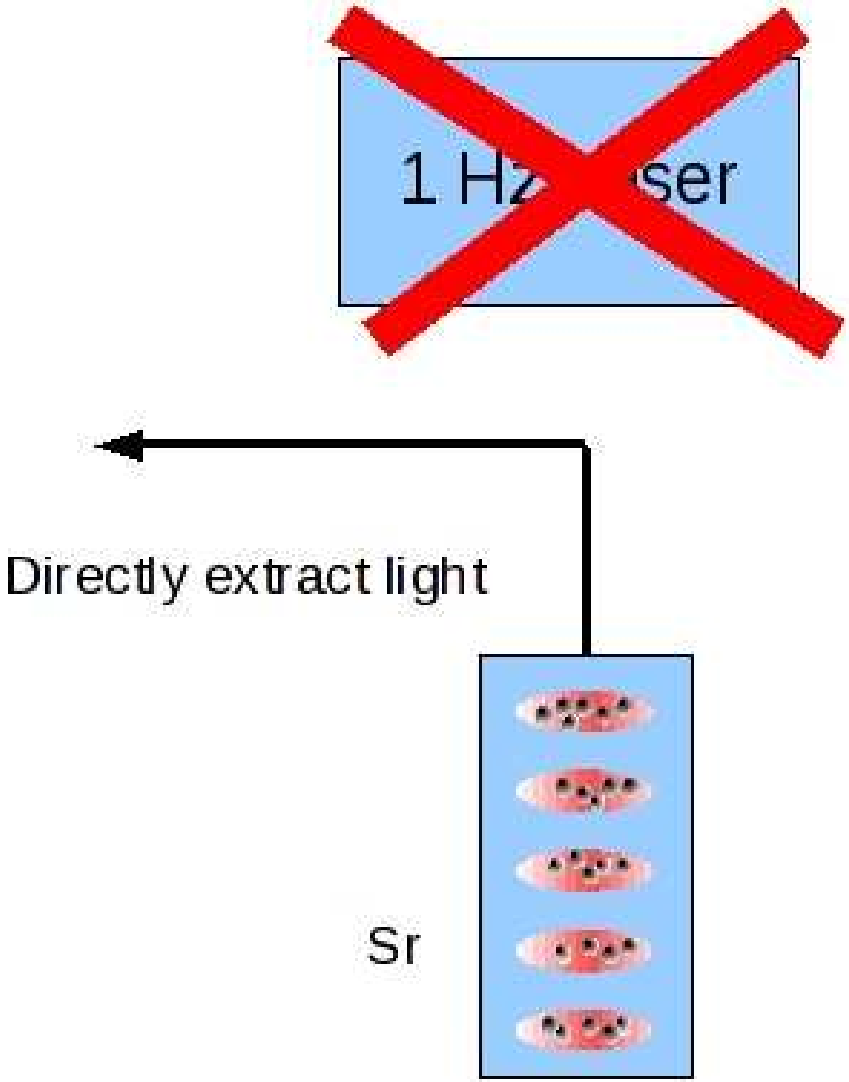
**Question**

Is it possible to extract radiation directly from the atoms (spontaneous emission)?

- Problem: Extremely feeble

$$P \sim N\Gamma\hbar\omega \approx 10^{-16}\text{W (in } 4\pi)$$

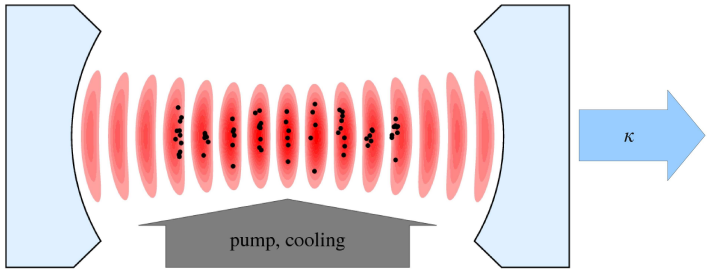
- Threshold of usefulness: $10^{-14}$ W



# Our proposal

Use collective emission inside a cavity to gain another factor of $N$

$$P \sim N^2 \Gamma \hbar \omega \approx 10^{-10}\mathrm{W} \text{ (in one mode)}$$

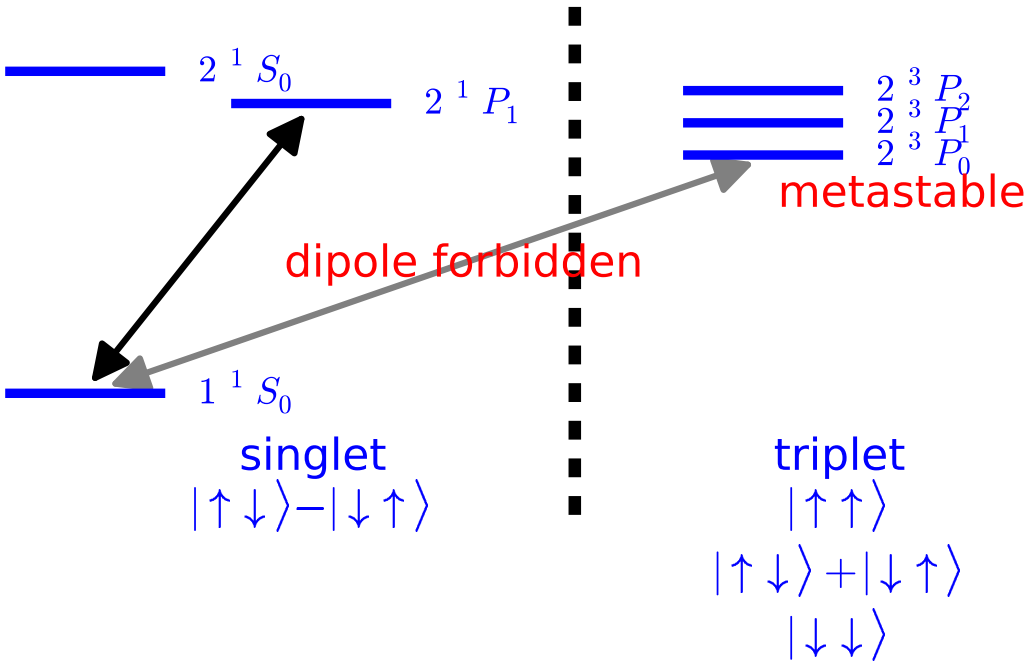



## Key message:

Forced to study cavity QED with extremely small dipole moment.

# Why group II atoms?

- Two-electron system
- Narrow intercombination lines
- Small dipole moment ($\wp \sim 10^{-5} e a_0$)

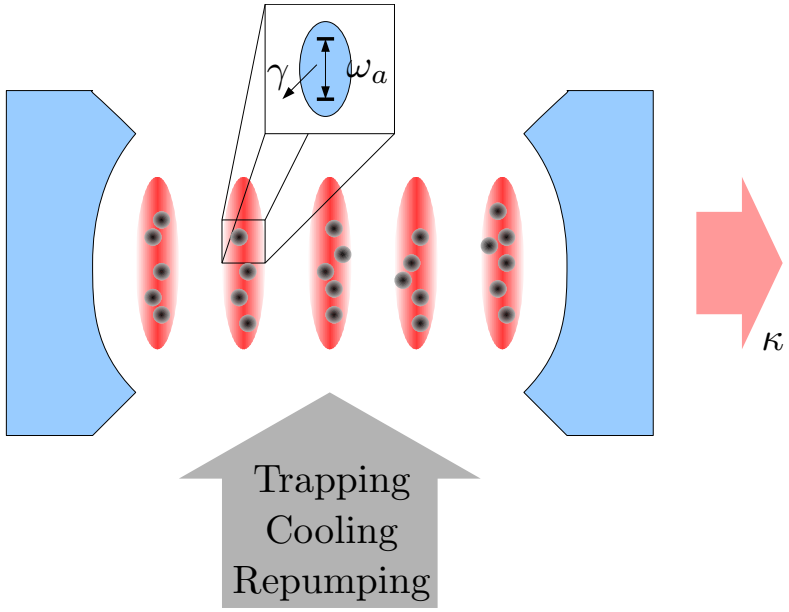



## Consequences:

- Weak coupling to cavity field
- Long atomic coherence times ($> 1$s)

# Model

- Two level atoms, in vibrational ground state (Lamb-Dicke regime)
- Single mode quantized light field
- Collective atom-field interaction
- Cavity decay (outcoupling)
- Non-collective decay processes (spontaneous emission, repumping, inhomogeneous broadening)



# Mathematical description

- Coherent part

$$\hat{H} = \frac{\hbar\omega_a}{2}\sum_{j=1}^{N}\hat{\sigma}_j^z + \hbar\omega_c\hat{a}^\dagger\hat{a} + \frac{\hbar\Omega}{2}\left(\hat{a}^\dagger\sum_{j=1}^{N}\hat{\sigma}_j^- + \hat{a}\sum_{j=1}^{N}\hat{\sigma}_j^+\right) .$$

- Completely collective

$$\hat{H} = \hbar\omega_a\hat{S}^z + \hbar\omega_c\hat{a}^\dagger\hat{a} + \frac{\hbar\Omega}{2}\left(\hat{a}^\dagger\hat{S}^- + \hat{a}\hat{S}^+\right) .$$

# Mathematical description, dissipative processes

- Von Neumann equation:

$$\frac{d}{dt}\hat{\rho} = \frac{1}{i\hbar}[\hat{H}, \hat{\rho}] + \mathcal{L}[\hat{\rho}]$$

- Liouvillian:

$$\mathcal{L}[\rho] = \mathcal{L}_{\text{cavity}}[\rho] + \mathcal{L}_{\text{spont.}}[\rho] + \mathcal{L}_{\text{inhom.}}[\rho] + \mathcal{L}_{\text{repump}}[\rho]$$

- For instance spontaneous emission:

$$\mathcal{L}_{\text{spont}}[\hat{\rho}] = -\frac{\Gamma}{2}\sum_{j=1}^{N}\left(\hat{\sigma}_j^+\hat{\sigma}_j^-\hat{\rho} + \hat{\rho}\hat{\sigma}_j^+\hat{\sigma}_j^- - 2\hat{\sigma}_j^-\hat{\rho}\hat{\sigma}_j^+\right),$$

- Decay processes address individual atoms (non-collective)

# Numbers

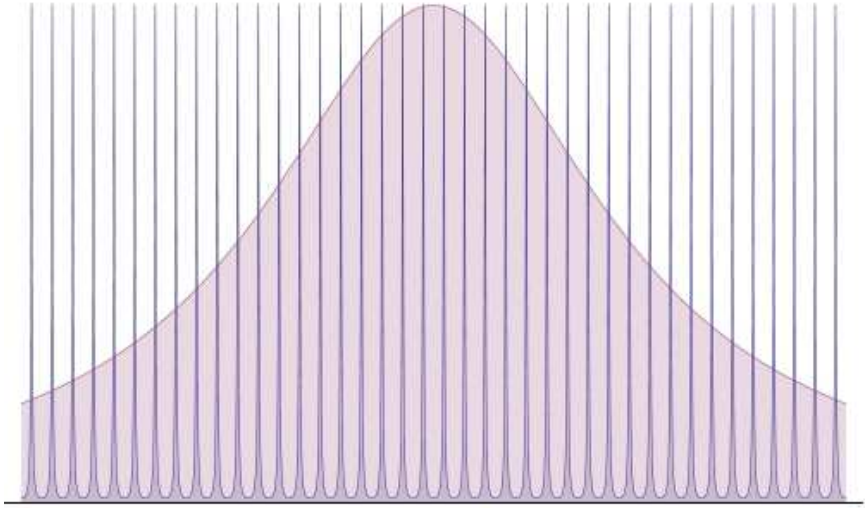
| parameter | symbol | value |
|---|---|---|
| homogeneous linewidth | $\Gamma$ | $0.01\ \mathrm{s}^{-1}$ |
| atom-field coupling | $\Omega$ | $37.0\ \mathrm{s}^{-1}$ |
| inhomogeneous lifetime | $T_2$ | $1\ \mathrm{s}$ |
| repumping rate | $w$ | $10^{-3} - 10^{4}\ \mathrm{s}^{-1}$ |
| cavity decay rate | $\kappa$ | $9.4 \times 10^{5}\ \mathrm{s}^{-1}$ |
| cavity finesse | $\mathcal{F}$ | $10^{6}$ |
| number of atoms | $N$ | $10^{3} - 10^{6}$ |
| cooperativity parameter | C | 0.14 |

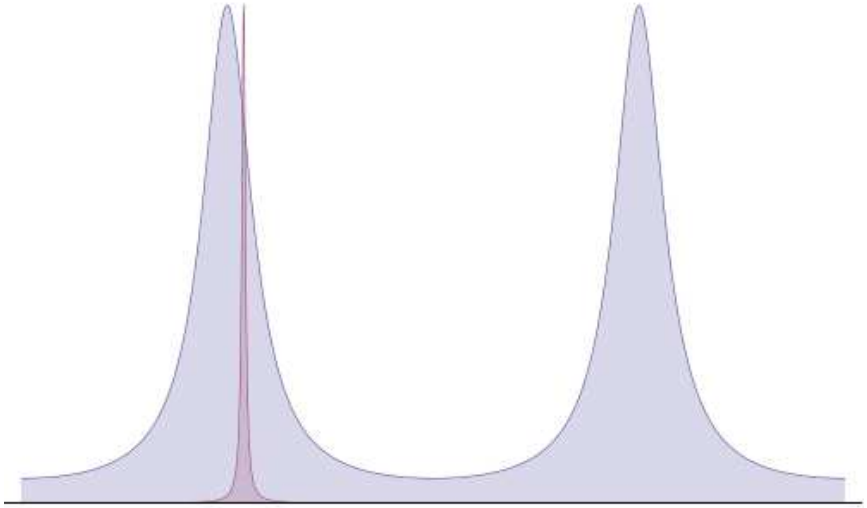
**Take home message:**

Cavity relaxation rate much faster than atomic relaxation rates

$C = \Omega^2/(\Gamma\kappa)$ (single atom cooperativity parameter)

# Bad cavity laser





- Normal laser: atomic relaxation rates much faster than field relaxation rates
- Here: atomic dynamics completely “contained” in bandwidth of cavity

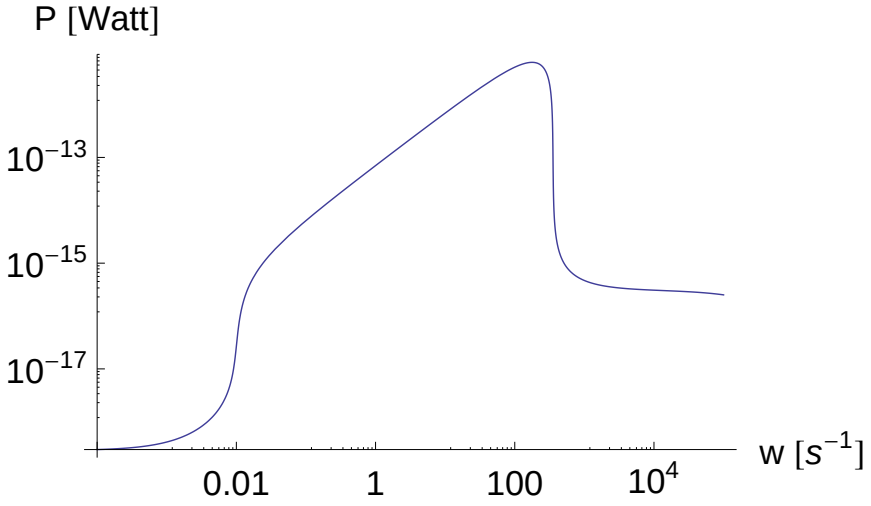
# Intensity

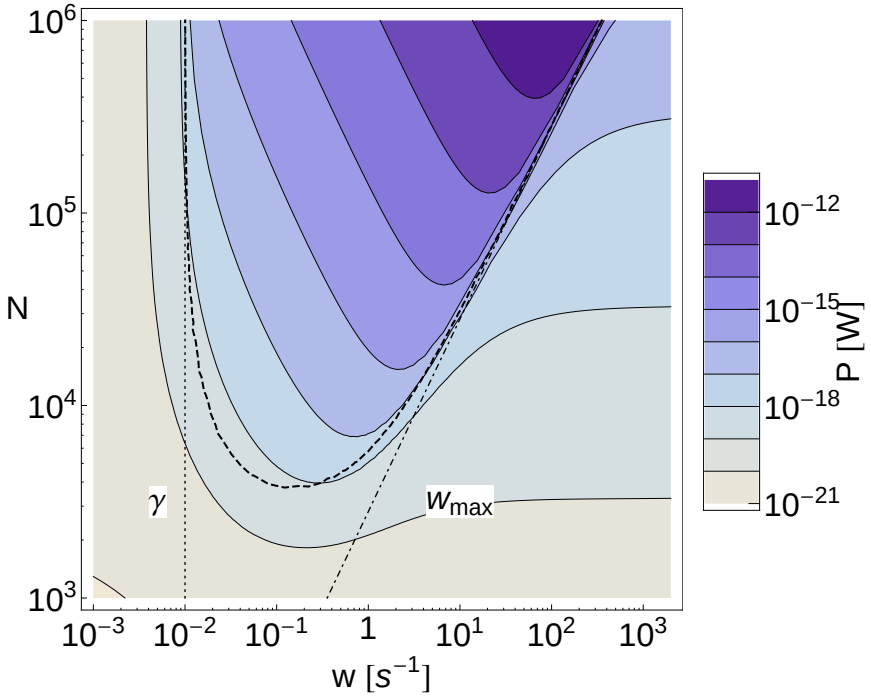
- Threshold: $w \sim \Gamma$
- Above threshold: sharp increase of emitted power
- Second threshold ($w_{\max} \approx NC\Gamma$): Non-collective emission
- Max. power:

$$P_{\max} = \frac{N(NC\Gamma\hbar\omega_a)}{8}$$

- Critical particle number:

$$N_{\text{crit.}} = \frac{4}{C\Gamma T_2}$$



# Spectra

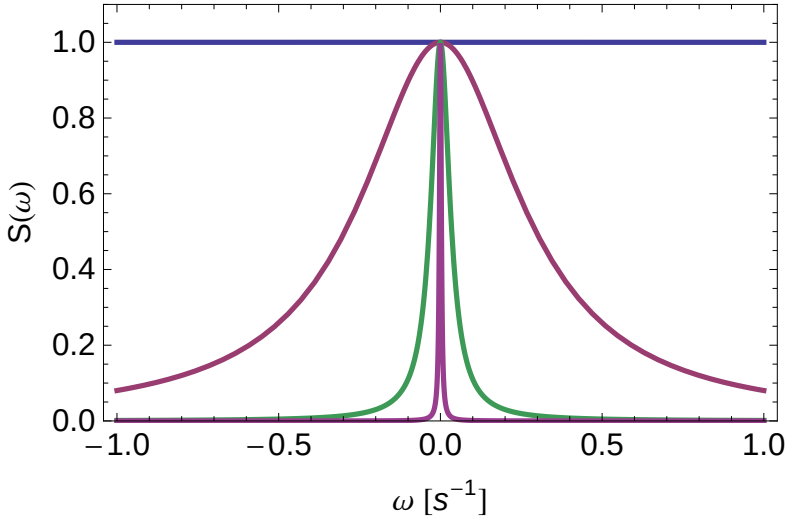
- Use quantum regression theorem to find $\langle \hat{a}^\dagger(t)\hat{a}(0)\rangle$
- Spectrum:

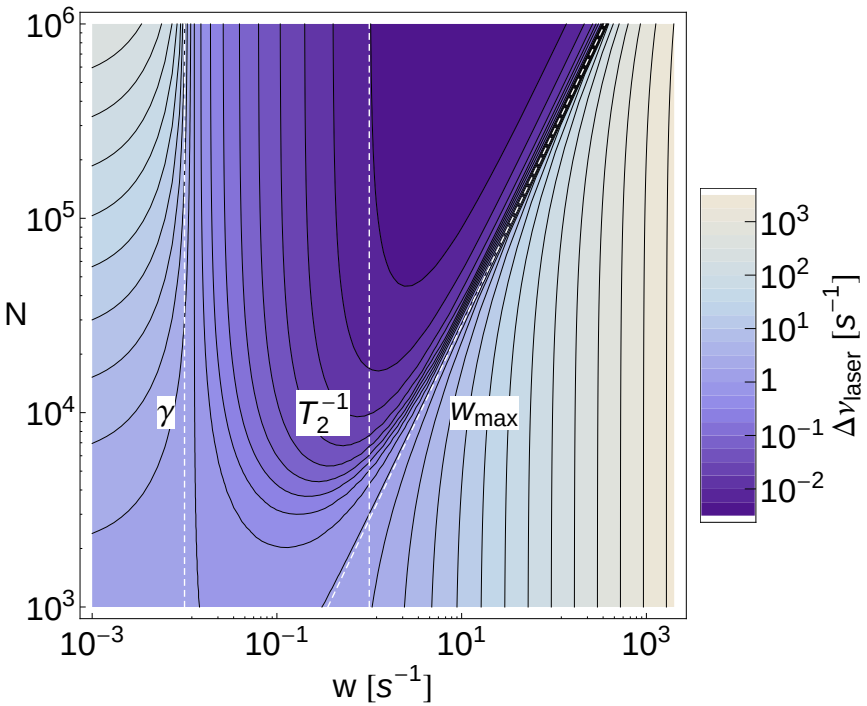
$$S(\omega) = \int_{-\infty}^{\infty} dt e^{-i\omega t} \langle \hat{a}^\dagger(t)\hat{a}(0)\rangle$$

- Minimum linewidth

$$\Delta\nu = 4C\Gamma$$

- Much smaller than Schawlow-Townes linewidth



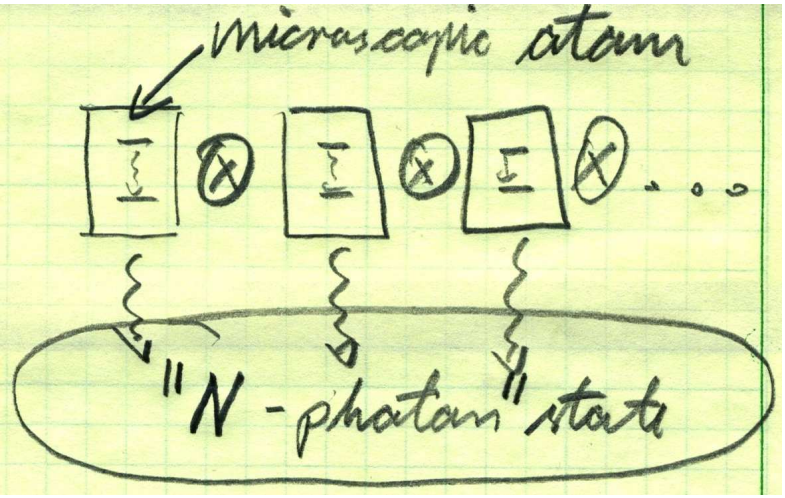



## Big picture question:

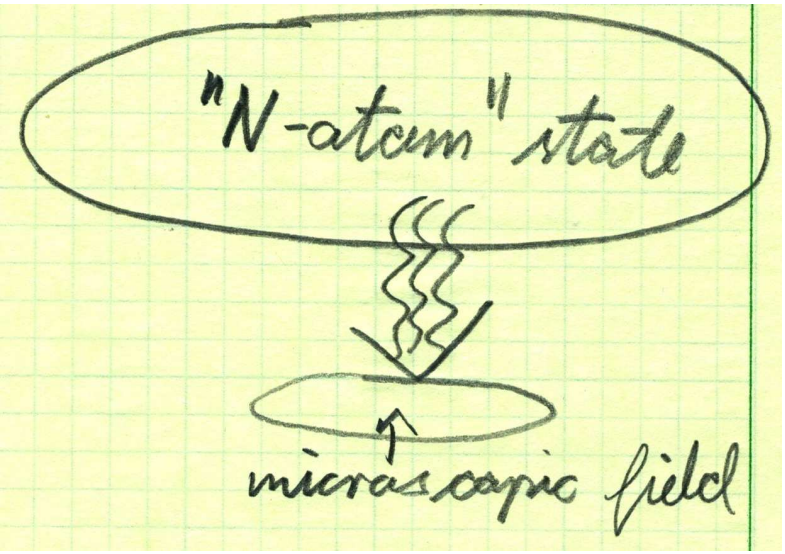
Isn't this just a laser?

# LASER vs. Superradiance

- Atoms: microscopic, independent
- Field: macroscopic, coherent
- Enhancement of emission due to stimulation
- Coherence due to final state




- Atoms: macroscopic, coherent
- Field: microscopic
- Enhancement of emission due to interference in initial state
- Coherence due to initial state



# Summary: physical realizations

- Lattice clocks
  - Extremely well controlled environment
  - Small decoherence rates
- Raman transitions between magnetic sublevels in Alkali-atoms
  - Powerful laser cooling schemes, evaporative cooling
  - Flexible level schemes (tunable coupling strengths, decay rates, . . .)
  - Several cavity QED experiments in existence
- Trapped ions
- NV-centers in diamond
  - In crystal $\Rightarrow$ easy to have many of them
  - Large inhomogeneous broadening
- **Nuclear** transitions in $^{229}$Th
  - Very insensitive to environment